EXPERIMENTAL STUDY OF PULSATING FLOW OF A POLYMER SOLUTION IN A CIRCULAR PIPE

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Analysis of a pulsating flow regime for a Newtonian liquid [1] shows that with identical average pressure gradients for pulsating and steady flow the average liquid flow rate does not change. Theoretical and experimental studies of non-Newtonian liquid flow have shown [2-11] that pulsation of the pressure gradient leads to a change in flow rate compared with the Newtonian case. This change is taken to be determined by the relative value $I = Q_p/Q_s - 1$, where Q_p and Q_s are average flow rate of pulsating and steady flow.

The aim of this work is to study the effect of external parameters (frequency, pulsation amplitude, and magnitude of the average pressure gradient) on the relative change in flow rate with a pulsating flow regime for concentrated solutions of high polymers in a circular pipe.

We carry out criterial analysis of a set of equations describing pulsating flow for a viscoelastic liquid in the section of steady flow. The equation of motion with a pressure gradient changing sinusoidally with time irrespective of the nature of the liquid has the form

$$\rho \frac{\partial u}{\partial t} = -\left(\frac{\partial p}{\partial z}\right)_c (1 + A \sin \omega t) + \frac{1}{r}\frac{\partial}{\partial r}(r\tau), \tag{1}$$

where A is pulsation amplitude; ω, pulsation frequency; ρ, liquid density; u, velocity; $(\partial p/\partial z)_c$, average pressure gradient. We use an expression for tangential shear stress τ following from the structural-phenomenological model for a nonlinearly viscoelastic medium [12]:

$$\tau = \varepsilon\left[\langle x_r x_z\rangle + \frac{\alpha\varkappa}{4}(\langle x_r x_r\rangle + \langle x_z x_z\rangle)\frac{\partial u}{\partial r}\right]; \tag{2}$$

$$\begin{gathered}\frac{\partial}{\partial t}\langle x_z x_z\rangle = \alpha\langle x_r x_z\rangle\frac{\partial u}{\partial r} - \frac{2}{\varkappa}(\langle x_z x_z\rangle - 1),\\ \frac{\partial}{\partial t}\langle x_r x_r\rangle = 2\langle x_r x_z\rangle\left(1 - \frac{\alpha}{2}\right)\frac{\partial u}{\partial r} - \frac{2}{\varkappa}(\langle x_r x_r\rangle - 1),\\ \frac{\partial}{\partial t}\langle x_r x_z\rangle = -\frac{\alpha}{2}\langle x_r x_r\rangle\frac{\partial u}{\partial r} +\\ + \langle x_z x_z\rangle\left(1 - \frac{\alpha}{2}\right)\frac{\partial u}{\partial r} - \frac{2}{\varkappa}\langle x_r x_z\rangle.\end{gathered} \tag{3}$$

Here ε is high elasticity parameter; κ, relaxation time; $\langle x_i x_j\rangle$, moments of the distribution function for nodes of the polymer network; α is kinetic stiffness of the polymer chain.

We introduce dimensionless variables: $t \to t/T$, $r \to r/R$, $\tau \to \tau/\tau_w$, $u \to u/V$, where $T = 2\pi/\omega$ is pulsation period; R, pipe radius; $\tau_w = 0.5R(\partial p/\partial z)_c$, tangential shear stress at the wall for a given average pressure gradient; $V = \tau_w R/\varepsilon_w \kappa_w$ is scale velocity of steady flow. If it is assumed that ε, κ, and α are not dependent on the velocity gradient, then in dimensionless form set of equations (1)-(3) takes the form

$$\mathrm{Re}_\omega \frac{\partial u}{\partial t} = 1 + A \sin 2\pi t + 0{,}5\,\mathrm{We}^{-1} r^{-1}\frac{\partial}{\partial r}(r\tau); \tag{4}$$

$$\tau = \langle x_r x_z\rangle + 0{,}25\alpha\,\mathrm{We}\,(\langle x_r x_r\rangle + \langle x_z x_z\rangle)\frac{\partial u}{\partial r}; \tag{5}$$

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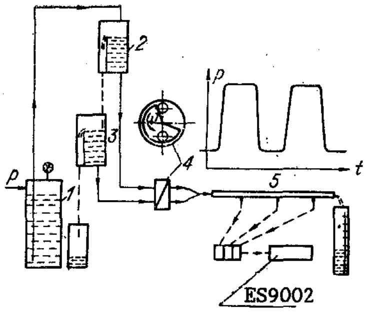

Fig. 1

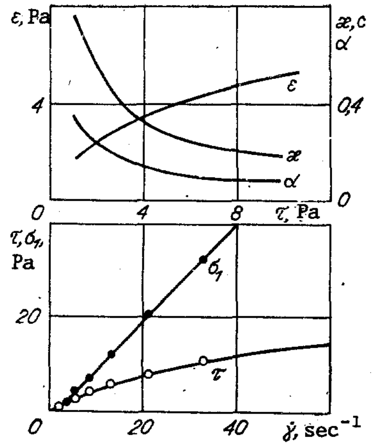

Fig. 2

$$\mathrm{De}\frac{\partial}{\partial t}\langle x_z x_z\rangle = \alpha\,\mathrm{We}\frac{\partial u}{\partial r}\langle x_r x_z\rangle - 2\left(\langle x_z x_z\rangle - 1\right), \tag{6}$$

$$\mathrm{De}\frac{\partial}{\partial t}\langle x_r x_r\rangle = 2\left(1-\frac{\alpha}{2}\right)\mathrm{We}\langle x_r x_z\rangle\frac{\partial u}{\partial r} - 2\left(\langle x_r x_r\rangle - 1\right),$$

$$\mathrm{De}\frac{\partial}{\partial t}\langle x_r x_z\rangle = -\frac{\alpha}{2}\mathrm{We}\frac{\partial u}{\partial r}\langle x_r x_r\rangle + \left(1-\frac{\alpha}{2}\right)\mathrm{We}\frac{\partial u}{\partial r}\langle x_z x_z\rangle - 2\langle x_r x_z\rangle.$$

Here complex $\mathrm{Re}_\omega = \rho R^2/2T\varepsilon_w\kappa_w$ is the ratio between time for passage of a shear wave across the pipe and the period of pressure gradient pulsation (oscillating Reynolds number); complex $\mathrm{We} = \tau_w/\varepsilon_w \equiv \kappa_w V/R$ is a derivative of relaxation time for typical shear velocity (Weissenberg number); $\mathrm{De} = \kappa_w/T$ is the relationship between typical relaxation time and the pressure gradient period (Debye number).

A change in parameters ε, κ, and α across the pipe section may be considered approximately in the form of complexes $\varepsilon_w/\varepsilon_0$, κ_w/κ_0, and α_w/α_0, where index 0 indicates a value of a parameter with $\tau \to 0$ (on the pipe axis). Then the relative change in flow rate will be a function of the following dimensional parameters:

$$I = f(\mathrm{Re}_\omega,\ \mathrm{We},\ \mathrm{De},\ A,\ \varepsilon_w/\varepsilon_0,\ \varkappa_w/\varkappa_0,\ \alpha_w/\alpha_0,\ \alpha_w). \tag{7}$$

The experimental equipment is an open loop (Fig. 1). The polymer solution from tank 1 under pressure is fed to constant level tanks 2 and 3, and then pulsator 4, which is a chamber with an obdurator. From the pulsator solution passes into the working section 5, i.e., a pipe 10 mm in diameter and 1100 mm long. The range of change in average pressure gradients is $2000 < -(\partial p/\partial z)_c < 9000$ Pa/m. The pulsation frequency $f = 1/T$ was varied in the ranges $0.15 < f < 1.8$ Hz. The amplitude of pulsation was specified by the difference in height between tanks.

The range of change in pulsation amplitude is $0.4 < A < 0.7$. Over the whole range of change in external parameters, the shape of the pulsating pressure signal $\partial p(i)/\partial z$ is close to rectangular (Fig. 1).

Measurement of pulsating and steady flow rate was accomplished by a volumetric method with an error of about 1%. The pressure gradient in the pipe was determined from the pressure value at the wall measured by strain gages set up along the channel length. The measurement accuracy for absolute pressure was ±0.2% of the nominal value. The frequency operating range for pressure sensors was from 0 to 7 Hz. The electric signal from the pressure sensor through an amplifier was recorded on an ES 9002 tape recorder. Information from the magnetic tape was treated in a computer.

TABLE 1

Fig. 4	τ_w, Pa	$100\mathrm{Re}_\omega$	10De	We	Fig. 4	τ_w, Pa	$100\mathrm{Re}_\omega$	10De	We
a	6,8	0,2—2,1	0,3—4	1,47	c	10,8	0,2—2,4	0,2—2,6	1,73
b	9,5	0,2—2,3	0,3—3	1,67	d	12,8	0,2—2,5	0,2—2,5	1,82

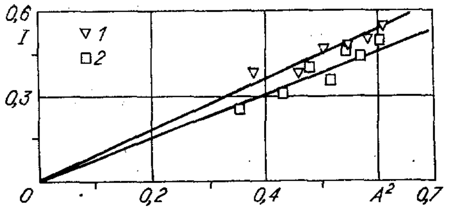

Fig. 3

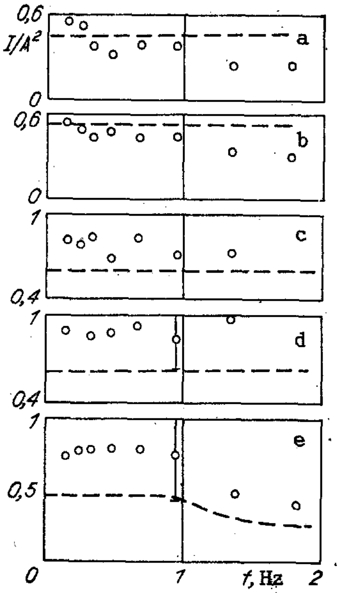

Fig. 4

A 1% solution of polyacrylamide (PAA) in water was used as the working liquid. The rheological characteristics of the solution were measured in an Istron-3250 rotation instrument. Values of ε, κ, and α (Fig. 2) were determined by the procedure in [12].

Given in Figs. 3 and 4 are experimental results for a steady pulsating flow regime of PAA solution in a circular pipe. The linear nature of the dependence of I on A^2 was previously determined theoretically [9-11] and confirmed experimentally up to a value of $A \leq 0.3$ [5-7]. Tests showed (Fig. 3) that this relationship is valid over the whole range of A occurring in our measurements. Points 1 in Fig. 3 were obtained with $\tau_w = 10.8$ Pa, and 2 with $\tau_w = 12.8$ Pa. Results are presented in Fig. 4a-d for measurement of I/A^2 values in relation to pulsation frequency f found with four different values of tangential stress. Dimensionless criteria for these regimes characterizing the flow process, determined with values of tangential shear stress at the wall, are presented in Table 1. It should be noted that, in spite of the significant amplitudes of pressure gradient pulsation and a satisfactory accuracy for measuring the pressure gradient and liquid flow rate, the smallness of the measured value of difference in flow rate in determining I/A^2 leads to a marked error in determining this value and the scatter of experimental data.

It can be seen that with an increase in tangential shear stress (Weissenberg number) there is an increase in the flow rate compared with the steady flow regime. A weak tendency toward a reduction in I/A^2 due to pulsation frequency of the pressure gradient may be detected with De = 0.3-0.4 (Fig. 4a, b).

Results presented in Fig. 4e were obtained for a PAA solution whose viscosity was lower by a factor of ~4. For frequencies exceeding 1 Hz the value of I/A^2 decreased sharply. The Re_ω increased to a value of the order of 0.1. It is possible that under these conditions a pulsating shear wave does not manage to penetrate into the core of the flow and the effect of pulsation on liquid flow rate decreases. Calculation carried out for a linear flow rule [11] (broken lines in Fig. 4) confirms this suggestion.

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GROUP PROPERTIES AND INVARIANT SOLUTIONS OF EQUATIONS DESCRIBING TWO-DIMENSIONAL FLOW OF GLACIERS

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One of the most important problems of contemporary glaciology is the construction of a mathematical theory of glacial mechanics, in which the development of mathematical models of glaciers plays a special role. Two different approaches can be distinguished in problems involving mathematical modeling of various processes. The first involves a tendency to construct a detailed model of the process under study, ensuring its adequacy by use of a large volume of experimental data, and then using the model to obtain quantitative conclusions and to apply such results in practice. The other approach involves construction of a spectrum of exact solutions for particular models, study of which would permit discovery of basic features of the process with less expenditure of time. Both directions are valid, and the results of the second can be used to justify and refine detailed mathematical models. The first approach was developed for glacial mechanics in [1-6], while the second has yet to enjoy such rapid growth. The results of the present study should be considered as a contribution toward the second approach toward mathematical modeling of glacial mechanics. In particular, the group properties of a nonlinear differential equation describing the position of the free surface of a glacier will be studied, invariant solutions of the equation will be constructed, and these solutions will then be used to study concrete problems arising in the study of glacier flow.

Considering the nonsteady state flow of a glacier in the isothermal approximation, it can be shown that the function $\ell(x, y, t)$ describing the free surface of the glacier satisfies a second-order nonlinear differential equation in partial derivatives

$$\begin{aligned}\frac{\partial l}{\partial t} = {} & \frac{\partial}{\partial x}\left\{\left[\frac{\partial l}{\partial x}\Big/\sqrt{\left(\frac{\partial l}{\partial x}\right)^2+\left(\frac{\partial l}{\partial y}\right)^2}\right]\int_{z_0}^{l}(l-z)\,\Gamma\left[(l-z)\sqrt{\left(\frac{\partial l}{\partial x}\right)^2+\left(\frac{\partial l}{\partial y}\right)^2}\right]dz\right\} + \\ & + \frac{\partial}{\partial y}\left\{\left[\frac{\partial l}{\partial y}\Big/\sqrt{\left(\frac{\partial l}{\partial x}\right)^2+\left(\frac{\partial l}{\partial y}\right)^2}\right]\int_{z_0}^{l}(l-z)\,\Gamma\left[(l-z)\sqrt{\left(\frac{\partial l}{\partial x}\right)^2+\left(\frac{\partial l}{\partial y}\right)^2}\right]dz\right\},\end{aligned}\tag{1}$$

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